RE-EXAMINING THE WORLD TRADE CENTER CATASTROPHE

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ABSTRACT. The Twin Towers of the World Trade Center collapsed in a progressive top to bottom manner on the 11th of September 2001 after they were struck by two aircrafts.

A model of a gravity-driven collapse of a tall building has been proposed by Bažant et al. We apply this model to the collapse of the North Tower to determine the energy dissipation per storey during the collapse. This has already been done by Bažant et al. for the first three seconds. Using video record data we extend this time range to over 9 seconds. Our findings are 250 MJ during the first 4.6 seconds. In the time interval between 4.6 and 7.7 seconds after collapse initiation we find an additional energy dissipation per storey of 2500 MJ. Because the steel columns increase in strength towards the ground this value corresponds to a value of 2000 MJ for the storeys in the aircraft impact zone. After 7.7 seconds the value reduces to the value that corresponds to the value during the first 4.6 seconds.

These results have two possible interpretations:

- (1) If due to the building design (column strength, shape etc.) the energy dissipation per storey cannot reach the high values which we observed, then the collapse cannot be described by the gravity-driven collapse model.
- (2) If the collapse is described correctly by the gravity-driven collapse model, then then we fond direct evidence that the collapse mechanism did not follow the same pattern during the whole of the collapse. The possible amount of energy dissipation was reduced by an order of magnitude during two long time time intervals.

In both cases there is no a priori reason to justify the sometimes expressed belief that the collapse was inevitable even after the falling top section had gained a significant amount of momentum. In fact, if the amount of energy dissipation had stayed only little longer on the high level, then a gravity-driven collapse would have arrested.

Note that (1) implies that if in principle the gravity-driven collapse model describes gravity-driven collapses of tall buildings, then the collapse was not gravitydriven.

Keywords: World Trade Center, North Tower, Progressive Floor Collapse, Crush-Down Equation, Energy Dissipation, Structural Dynamics, High-Rise Buildings, New York City, Terrorism.

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1. INTRODUCTION

1.1. **The Case.** On the 11th of September 2001 three major buildings collapsed in New York City. They were part of the World Trade Center complex which consisted of seven buildings overall. The focus in this paper lies on one of the three. It was called the North Tower of the World Trade Center. At the time it was built it was the tallest building in the world with a height of 417 m, 110 storeys and a huge antenna on top. In the morning of the 11th of September 2001 it was struck by an aircraft. The fuselage of the aircraft impacted on the height of the 96th storey roughly 50 m below the top. The whole building collapsed 102 minutes later [NIST, 2005a, Ch.2].

1.2. An Attempt for Explanation: The Gravity-Driven Collapse. An American government agency, the National Institute of Standards and Technology, issued a report in 2005 that tried to explain how the collapse initiated [NIST, 2005a]. However, they did not target the question how the collapse progressed. Two years later, in 2007, a model was proposed by Bažant and Verdure that describes the collapse of the North Tower as a gravity-driven progressive collapse [Bažant and Verdure, 2007].

Therein the collapsing building is modelled by three distinct parts which are: 1. The initial top section that sat above the first failing floor (this section keeps its height until the crushing front hits the ground). 2. The section below the top section which is compacted from its original undamaged size and moving with the same velocity as the top section (the height of this section is growing in time). 3. The resting, still undamaged section below these two (the height of this section is reducing).

During the course of the collapses the initial top section stays undestroyed and the height of the falling section (the top and the middle section together) is strictly increasing until the crushing front reaches the ground. Then the top section is destroyed. It should be emphasised that this behaviour—the undestroyed top section—is not a choice of model parameters but a consequence of the underlying Newtonian equation of motions. The argument for this conclusion is sketched in [Bažant et al. 2008, Appendix] where a two-sided front propagation is computed.

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FIGURE 1. Schematic illustration of the assumed collapse of the North Tower.

The upward directed crushing front stops within a fraction of a second after having propagated an extremely short distance only.¹ So up to this initial and neglectable decay the height of the falling section must increase.

The amount of how much the falling section is growing in height is specified by how much the middle section is compacted. When the crushing front reaches a storey, the ratio of the full height of that storey divided by the height after the crushing front has passed is called the compaction parameter $\lambda \in [0, 1]$. In [Bažant and Verdure, 2007] it is assumed that all storeys are compacted to the same height, and a value of $\lambda = 0.18$ is used. So if $\kappa_{out} \in [0, 1]$ is the parameter that specifies the fraction of material that is spit outwards during the collapse at the crushing front, then

(1)
$$\lambda = (1 - \kappa_{\text{out}}) V_1 / V_0,$$

where V_0 is the initial volume of the tower, V_1 is the volume of the compacted rubble pile of the tower. In the numerical analysis of [Bažant and Verdure, 2007] $\kappa_{out} = 0$ is used, in [Bažant et al. 2008] a value of $\kappa_{out} = 0.2$ is considered to be reasonable. The actual value of κ_{out} does effect the downward movement only gradually (cp. Figure 4). In any case, if the crushing front has propagated a certain distance, then the height of the falling section has increased by λ times this distance.

Now let us fix a coordinate system which is pointing downwards to the ground and whose origin has a fixed elevation above concourse level, namely the elevation of the initial undestroyed tower top (cp. Figure 1). Let $z_0 > 0$ be the position of the storey that collapsed first at the time of collapse initiation (t = 0), i.e. z_0 is the height of the undestroyed top section. If $z(t) > z_0$ is the position of the crushing front at time t > 0, then $z(t) - \lambda(z(t) - z_0) - z_0 = (1 - \lambda)(z(t) - z_0)$ is the position of the roof top at time t, and its time derivative $(1 - \lambda)\dot{z}(t)$ is the

¹ Unfortunately, the authors do not comment why they include the term $\frac{m_2}{2}\dot{x}$ to the momentum of the top section in (33) of [Bažant et al. 2008] or (4) of [Bažant and Le, 2008]. A term like this could be considered in (32) as a result of the first colliding storeys. In (32) this term would avoid a vanishing mass of the compacted layer at time 0 if the initial conditions x(0) = z(0) are used. (A non-vanishing mass is needed to solve the equation for \ddot{z} .)

downward velocity of both the top and the middle section. Therefore the total momentum of the falling two sections is given by $p(t) = m(z(t))(1 - \lambda)\dot{z}(t)$, where

(2)
$$m(z) := \int_0^{z_0} \mu(x) \, dx + (1 - \kappa_{\text{out}}) \int_{z_0}^z \mu(x) \, dx$$

describes the accumulated mass of the two moving sections. $\mu(\cdot)$ is the mass height-density of the undestroyed tower.

Then the equation of motion—which is called Crush-Down Equation in [Bažant and Verdure, 2007]— that is valid until the crushing front reaches the ground is given by

(3)
$$\frac{d}{dt}\Big(m\big(z(t)\big)(1-\lambda)\dot{z}(t)\big) = m\big(z(t)\big)g - F\big(z(t)\big),$$

where $F(\cdot) > 0$ is the upward resistance force due to column buckling, and *g*, evidently, is the acceleration of gravity in New York City.

To model the aircraft impact damage and the fire damage of the tower let $\chi(z) \in [0,1]$ be the parameter which specifies how much the columns are weakened at *z*. $\chi(z) = 1$ means full support. So the upward force *F* is the product $F = \chi \cdot F_0$, where F_0 describes the undamaged column force. For our numerical analysis we will use

(4)
$$\chi(z) = \begin{cases} 0.5, & \text{for } z \in [z_0, z_0 + h) & (\text{first failing storey}), \\ 0.9, & \text{for } z \in [z_0 + h, z_0 + 4h) & (\text{impact zone, cp. Fig. 16}), \\ 1, & \text{for } z \ge z_0 + 4h & (\text{intact building}). \end{cases}$$

The shapes of μ and F_0 are specified in [Bažant et al. 2008, Fig. 2(a)] essentially as piece-wise linear functions, where the slope of the linear increasing part of F_0 is chosen proportional to the (increasing) cross-sections of the columns:²

(5)
$$\mu(z) = \mu_0 \cdot \begin{cases} 1, & \text{for } z \in [z_0, 29h), \\ 1 + 0.43 \cdot \frac{z - 29h}{H - 29h}, & \text{for } z \ge 29h, \end{cases}$$

(6)
$$F_0(z) = \frac{W}{h} \cdot \begin{cases} 1, & \text{for } z \in [z_0, 29h], \\ 1 + 6 \cdot \frac{z - 29h}{H - 29h}, & \text{for } z \ge 29h, \end{cases}$$

where $\mu_0 = \mu(z_0)$ is a constant, *h* is the height of one storey, *H* the height of the tower, and *W* is the maximal energy absorption capacity of the buckling columns per storey at the height of the aircraft impact.

Of course, in a realistic scenario the force F_0 is not piece-wise linear, and the model will give unphysical solutions if the parameters are close to collapse but in reality still stable. One could add to F_0 a periodic function with period h and vanishing integral to enhance the model to get rid of these unphysical solutions. However, *if* collapse occurs, then only the average over one storey is the energet-ically relevant quantity. In this case F_0 should be regarded as the average upward force, and the error with respect to the enhanced model is tiny.

1.3. **The Magnitude of Energy Dissipation.** The main goal of this paper is to determine the quantity *W* under the premiss of a progressive floor collapse of the North Tower as described by the Crush-Down Equation.

In [Bažant and Zhou, 2002] and [Bažant et al. 2008] a maximal possible value of W = 500 MJ is mentioned, which is based on computations for a three-hinge

² It is mentioned on pages 895, 896 of [Bažant et al. 2008] that the transition into the linear increasing part should happen at the 81 storey for $\mu(\cdot)$ and $F(\cdot)$. Therefore the term 29 *h* appears. The building had 110 overground storeys.

buckling scenario. Yet meanwhile Korol and Sivakumaran have made empirical studies of buckling columns, which indicate that this value should be about 3 to 4 times bigger [Korol and Sivakumaran, 2014]. In [Bažant and Le (2016)] it is suggested that this value again should be corrected by a factor of $^{2}/_{3}$. Taking these considerations seriously, a value of W = 1000 MJ up to W = 1300 MJ or maybe even more might be considered as realistic if the collapse mechanism is is based on the three-hinge buckling scenario. Note that in [Szulandziński et al., 2013] a value of even 2700 MJ is proposed.

It should be emphasised that these values do not take any empirical data into account that come from the actual collapsing tower. A first account on this has been achieved by analysing video record data from the first three seconds of the collapse in [Bažant et al. 2008, p. 902], where an average upward force due to column buckling of 0.1 GJ/m has been reported. This correspond to a value of energy dissipation of W = 380 MJ per storey (h = 3.8 m).

Using a larger pool of video record data we shall extend the empirical range to over 9 seconds in total. We observe a slightly smaller value of about 250 MJ for the time period of 4.6 sec. However, we find that this value cannot stand the empirical data in the three second time interval between 4.6 sec and 7.7 sec, where we shall find an additional value of more than 2500 MJ of energy dissipation per storey. Taking the increasing strength of the columns into account, this value corresponds to a value of more than W = 2000 MJ at impact level.

This amount of energy dissipation would—if it lasted longer—arrest the fall within the next 10 meters. However, after that period of three seconds the energy dissipation per storey reduces again to the initial low value (relative to the value at impact level).

We do not speculate how it was possible that the energy dissipation rises and decays by an order of magnitude, but do point out that this was the case. A thorough investigation of the collapse mechanism needs to be done in order to understand how such an extreme difference of energy dissipation over long time intervals was possible.

We conclude in particular that there is no a priori reason that one should unconditionally assume that the columns of the building were designed too weak to arrest the fall even after the falling top section had gained a significant amount of momentum.

1.4. **The Modified Model.** In [Bažant et al. 2008] the derived Crush-Down Equation is modified on the left-hand side as well as on the right-hand side. Let us discuss and clarify these modifications. We start with the left-hand side.

- (lhs 1) The compaction parameter is supposed to increase proportionally with $\mu(\cdot)$. I. e. instead of assuming that every storey is compacted to the same height, it is assumed that every storey is compacted to the same density. We do not feel convinced that this necessarily more realistic, because it seems reasonable to expect that during the collapse the lower storeys are compacted to a higher density than the storeys above. In any case, this is only a tiny modification, and for simplicity we will ignore it and take $\lambda = const$ in what follows.
- (lhs 2) The velocity profile of the middle section is supposed to be non trivial. It is assumed to vary linearly from the top of the middle section down to the crushing front. However, this modification is not done accurately in [Bažant et al. 2008] for the following reasons:
 - (a) If the velocity profile is non trivial, then conservation of mass implies that the density of the compacted section is also varying. Yet in [Bažant et al. 2008] it is assumed that the density is constant.

(b) The linear velocity profile of [Bažant et al. 2008] is assumed to vary between the velocity of the top section (at the top of the compacted layer) and the velocity of the crushing front (at the bottom of the compacted layer). This is an extremely unphysical assumption, because the latter velocity is bigger than the first one. Realistically, the velocity at the bottom of the compacted layer should be lower than the velocity at the top. The velocity of the crushing front should not be regarded as the velocity of any mass-bearing instance, but as a quantity that describes the change of the geometry of the crushing building.

The interested reader is advised to have a look at [Schneider, 2017b], where detailed account of how to deal with non-trivial velocity profiles in the compacted section is given. Therein a version of the Crush-Down Equation for a rather general class of non-trivial velocity profiles is derived for both cases $\lambda = const$ and $\lambda \sim \mu$. The result is that the modified left-hand side of the Crush-Down Equation in [Bažant et al. 2008] is not only based on unphysical assumptions, but also that the resulting modification has the wrong sign, and under realistic assumptions its absolute value is far too big. The wrong sign and the wrong absolut value also changes the solution of the Crush-Down equation in the wrong way [Schneider, 2017b, Figure 3]. This can partially explain why during the first seconds the observed energy dissipation in [Bažant et al. 2008] is reported by 100 MJ, whereas we find a bigger value of about 250 MJ.

In any case these adjustments are small, and for simplicity we ignore these technicalities here: We do not make any changes on the left-hand side of the Crush-Down Equation.

Let us now turn to the modifications on the right-hand side. The upward resistance force—which in (3) is supposed to be the force due to column buckling only—is completed by three other terms. They originate in the pulverisation of the concrete floor slabs, the kinetic energy of the ejected air in the squeezed storeys and the kinetic energy of the solid ejected material ($\kappa_{out} \neq 0$).

(rhs 1) The term due to ejection of solid material from the tower is derived from the assumption that a certain fraction $\kappa_e \in [0, 1]$ of all the outwards-thrown material is kicked out at the crushing front with the velocity of the falling section, and the other fraction of material has vanishing velocity. This implies that the term

$$\underbrace{\frac{1}{2}\kappa_e\kappa_{\text{out}}(1-\lambda)^2}_{=:\alpha}\mu(z)\dot{z}^2.$$

should be added to the upward Force *F*. Note that in [Bažant et al. 2008] the factor $(1 - \lambda)^2$ does not appear as the bottom of the compacted layer is assumed to move with the velocity of the crushing front. Of course, this factor can be suppressed by rescaling κ_e . A value of $\kappa_e = 0.2$ is used in [Bažant et al. 2008].

(rhs 2) Once the crushing front has passed, the air inside a crushed storey got ejected. This causes an additional term

 $\beta \cdot \dot{z}^2$

that should be added to the upward force. Here no term $(1 - \lambda)^2$ appears, as the ejection of air is due to the geometric changes of the collapsing building, which happen with velocity \dot{z} (and not with the velocity of the falling section). For the precise structure of β see [Bažant et al. 2008, p. 897], where a numerical range of β from approximately $40 \cdot 10^3 \text{ kg/m}$ to $100 \cdot 10^3 \text{ kg/m}$ is

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derived. The higher values of β seem to be rather artificial, as the air in the building might also escape through the elevator shafts and through the brocken floor slabs of a collapsing storey. This is has not been taken into account in [Bažant et al. 2008]. We will use a value of $\beta = 50 \cdot 10^3 \text{ kg/m}$.

(rhs 3) For the pulverisation of concrete another term is brought into the Crush-Down Equation:

$$\underbrace{\frac{\gamma \cdot \frac{m_c}{2h}}{=:\beta'}}_{=:\beta'} \cdot \dot{z}^2,$$

where γ is a constant and m_c is the mass of the concrete floor slabs. The numerical value used in [Bažant et al. 2008] is $\beta' = 55 \cdot 10^3 \text{ kg/m}$.

No clear explanation is given in [Bažant et al. 2008] why this term should occur: The Crush-Down Equation expresses the change of momentum under a continuous series of collisions. The total momentum after impact is not effected if the colliding objects break into pieces or if they stay intact.

In any case, this term has the same structure (*const* $\cdot \dot{z}^2$) as the term in (rhs 2), so the general structure of the Crush-Down Equation does not change if the term is included or not, and we will only refer to β .

To summarise the modifications on the right hand side: A term of the form $-(\alpha \mu(z) + \beta) \dot{z}^2$ should be added.

1.5. **The Downward Movement (Part 1).** For the numerical analysis let us transform the Crush-Down Equation, which is a 1-dimensional differential equation of 2nd order, into its corresponding 2-dimensional equation of 1st order. Firstly, it can be rewritten as

(7)
$$\ddot{z} = \phi(z) - \psi(z) \dot{z}^2,$$

where

(8)
$$\phi(z) = \frac{g}{(1-\lambda)} - \frac{F(z)}{(1-\lambda)m(z)}$$

$$\psi(z) = \frac{(1-\kappa_{\text{out}})\mu(z)}{m(z)} + \frac{\alpha \mu(z) + \beta}{(1-\lambda) m(z)}.$$

Secondly, if $(z, u) \mapsto X(z, u)$ is the vector field given by

(9)
$$X(z,u) := \begin{pmatrix} u \\ \phi(z) - \psi(z) u^2 \end{pmatrix},$$

we shall consider the equation $\frac{d}{dt}(z, u) = X(z, u)$, which is equivalent to the original Crush-Down Equation. To analyse this equation numerically we use the open source computer algebra system Maxima (wxMaxima 16.04.0, [Maxi]) that is equipped with a pre-implementation of the Runge-Kutta algorithm. The source code is given in Appendix C. Figure 2 shows the height of the tower top as a function of time as derived from the Crush-Down Equation. The solutions in the left diagram are computed for the following choice of parameters:

(10)

$$\beta = 0.05 \cdot 10^{6} \text{ kg/m}, \quad H = 417 \text{ m}, \quad z_{0} = 46 \text{ m}$$
 (cp. Section 2.3),
 $\kappa_{e} = 0.2, \quad h = 3.8 \text{ m}, \quad \dot{z}(0) = 0,$
 $\mu_{0} = 0.6 \cdot 10^{6} \text{ kg/m}, \quad g = 9.8 \text{ m/sec}^{2}.$



FIGURE 2. Solutions of the Crush-Down Equation (converted into the elevation of the roof by $z \mapsto H - (1 - \lambda)(z - z_0)$) for two different masses of the tower. The left diagram was computed for $\mu_0 = 0.6 \cdot 10^6 \text{ kg/m}$, the right one for $\mu_0 = 0.9 \cdot 10^6 \text{ kg/m}$.

For comparison we have included the right diagram with a total mass of the tower increased by 50%. For $\mu_0 = 0.57 \cdot 10^6 \text{ kg/m}$ the total mass of the tower (including 21 m of underground storeys [NIST, 2005b, p. 19]) is 288,000 t. This value has been estimated meticulously in [Urich, 2007]. In [Bažant et al. 2008] a value of 500,000 t is stated without reference, which would give $\mu_0 = 0.98 \cdot 10^6 \text{ kg/m}$.

2. LENGTH MEASUREMENTS OF THE NORTH TOWER'S COLLAPSE

2.1. **The Idea of Measurement.** Our plan is to analyse video footage from different records of the North Tower's Collapse. The sources are a short film documantary by Etienne Sauret called *24 Hours* [Sauret] a History Channel documentary called *The 9/11 Conspiracies: Fact or Fiction* [History Channel] and some footage from CBS [CBS] and CNN [CNN]. Our goal is to determine the position of the roof under the principal assumption that the initially falling top section of the building stays undetroyed during the course of the collapse. As we have mentioned this principal assumption is a consequence of a gravity-driven collapse.

During the first three and a half seconds the top section is visible in Sauret's video record, which enables a direct measurement of the height of the roof. After the roof disappeared behind the dust cloud the antenna is still visible, so we can trace the roof by tracing the movement of the antenna.

After the antenna disappeared we can still make reasonable statements about the position of the roof by just estimating the crushing front from below. This is done with the video clips of History Channel and CBS. The initial height of the top section plus the height of the compacted section must be added to the measured lower bound of the crushing front to obtain a lower bound for the position of the roof.

2.2. Video Analysis Tool and Machine Data. We do some simple length measurements with the open-source video analysis tool *Tracker*, Version 4.96 [Trac], running on a 2.7 GHz Intel Core i5 iMac with operating system OSX 10.11.3 (15D21). It is equipped with an 8 GB 1600 MHz DDR3 RAM, and an Intel Iris Pro 1536 MB graphics card.



FIGURE 3. Horizontal calibration at frame 800, t = -4.47 sec. (For a zoom on the measurements go to Figure 11.)

2.3. **Etienne Sauret.** We use a sequence of stills from the short film 24 *Hours* shot by Etienne Sauret [Sauret] to determine the elevation of the top of the tower at three different times after collapse initiation.

Time is always measured relative to the collapse initiation at t = 0, which for us is the first recognisable movement of the north-west corner of the roofline. In the video copy we use this happens at frame number 934 (first frame has number 0). The first visible movement of the antenna is three frames earlier. The frame rate of the video is 29,97 frames per second, i. e. 3 frames in 0.10 seconds, which means the uncertainty in time is about 0.033 sec.

Let us now chronologically follow our measurements.

Figure 3 shows frame 800, i.e. t = -4.47 sec. It shows a foreground building in the left. All other images are cropped and only show the right part of the actual video image. The foreground building is 101 Avenue of the Americas (6th Avenue). The video was shot from 145 Avenue of the Americas which is an 8 storey building (s. building description in Figure 12 and Figure 13), where Sauret's film company Turn of the Century Pictures was based on the 7th floor [Turn]. This position gave an almost orthogonal perspective on the north side of the North Tower with a distance of $(1,550 \pm 20)$ m (Figure 12). The roof of the North Tower (without its antenna) had an elevation of 1,368 ft = 417.0 m [NIST, 2005a, p.5]. The optical center of the camera is targeting approximately 30 m below the roofline slightly to the east of the building. After estimating the height of the camera with another $30 \text{ m} \pm 10 \text{ m}$ we obtain an upward camera angle of $\arctan((417-60\pm10)/(1550\pm20)) = 13^{\circ}\pm1^{\circ}$. The sideward camera angle is estimated by $6^{\circ} \pm 1^{\circ}$ (Figure 12). Therefore, measured vertical distances have to be scaled up by a factor of $1/(\cos((6\pm 1)^\circ)\cos((13\pm 1)^\circ)) = 1.03 \pm 0.01$ when compared with horizontal distances on the north face of the tower. (The camera's angle of view is small and neglected). Below we shall make an explicit comparison with a known vertical distance.

The horizontal length calibration of the video is done as follows: Seven times we have measured the distance of thirty columns including the gap to the next column, where we have set one of the distances to a reference scale of 100 units. Figure 3 shows these measurements. The yellow line is the reference line. Figure 11 shows a zoom on the relevant part of the image. From top to bottom these seven measurements are:

99.99, 98.96, 98.94, **100**, 99.10, 102.2, 99.82.

The mean is 99.86 with a standard deviation of 1.05. The structural diagrams of the steel segments used in the construction of the tower are shown in [NIST, 2005b, p. 25]. The width of a segment of three columns (including the gap to the next column) is stated as 10 ft 0 in, so thirty columns and the gap to the next one had a width of 100 ft 0 in = 30.48 m. Therefore we will use

(11)
$$(99.86 \pm 1.05)$$
 reference units = (30.48 ± 0.32) m.

Our baseline for vertical length measurements is the horizontal line touching the north-west corner of the roof. This is the slightly thickened line of the grid in Figure 3. From Sauret's camera perspective the north-east corner appears to be approximately 1 m lower than the north-west corner.

We should now compare the horizontal calibration with a known vertical length. This is done by the measurement in Figure 14. The white line indicates the horizontal calibration of 30.48 m. The yellow measurement from the baseline to the red line gives a vertical distance of $(1.03 \pm 0.01) \cdot (60.9 \pm 0.6)$ m = (62.7 ± 1.3) m. We can identify the red line with the long white line in Figure 15, which itself can be identified with the 95th floor (Figure 16). According to the structural drawings of the tower, the distance from the roofline to the 95th floor was 90 ft 1 in = 63.73 m (Figure 17). So the deviation is within our range of precision and we can proceed.

Three meters below the roofline the visible end of the steel columns appears as a transition from the lighter roof to the darker lower side of the building. We refer to this line as the 'bottom of the roof'. It is sometimes easier to identify than the roof itself. Figure 18 shows the collapse initiation at frame 934. We measure 10 m away from the corners the position of the bottom of the roof. We find it $1.03 \cdot 4.2 \text{ m} = 4.3 \text{ m}$ and $1.03 \cdot 3.0 \text{ m} = 3.1 \text{ m}$ below the baseline. So in the middle we have a distance of 3.7 m to the baseline, i. e. in the middle the top of the roof has a distance of 0.7 m to the baseline

Figure 19 shows frame 957, t = 0.77 sec. The middle one of the three yellow arrows points from the top of the roof to where one might think the collapse initiated at the north-west corner of the building. This is the middle of the lighter part of the appearing dust cloud. It has a length of 1.03 · 44.76 m = 46.1 m. The arrow in the left indicates a part of the perimeter columns which move simultaneously with the top section: As far as one can say, at this stage the crushing front is no clean horizontal line. The yellow line to the right has a height of $z_0 = 1.03 \cdot 77.7 \,\mathrm{m} = 80 \,\mathrm{m}$, which is the height that is used in [Bažant and Verdure, 2007, Fig. 6] for the initially falling block. Clearly, this is an overestimation of that height. It has already been pointed out in [Szulandziński et al., 2013] that the mass of the falling block has been overestimated in [Bažant and Zhou, 2002, Bažant and Verdure, 2007, Bažant et al. 2008]. The wrong height assumption is probably the origin of this error, because the values match the mass distribution functions given in [Bažant and Verdure, 2007, Fig. 6]. Note that in [NIST, 2005a, p. 151] it is mentioned that the collapse initiated at the 98th floor. According to the structural drawings (Figure 17) the 12 storeys above had a height of 162 ft 1 in = 49.6 m. We will use therefore use $z_0 = 46$ m as a *lower bound* for the height of the initially falling block.

The red line in Figure 20 has an angle of 2.4° . It shows frame 983, t = 1.64 sec. We find the top of the roof at $1.03 \cdot (12.7 - 3) \text{ m} = 10.0 \text{ m}$ below the baseline (the green measurement line).

The antenna had clearly recognisable sections that appear white and dark from the front perspective. At frame 1024, t = 3.00 sec, the bottom of a white part is visible at the top of the video. The measured distance between the lowest point of the white part of antenna and the bottom of the roof is 60.4 m. This is the light blue line in Figure 21. The antenna has an angle of approx. 2° to the east at this time. (The antenna's angle to the south reaches a value of 8° before it is not visible any more. See below.)

The red line in Figure 22 has an angle of 3.7° . It shows frame 1030, t = 3.20 sec. We find the top of the roof at $1.03 \cdot (36.8 - 3)$ m = 34.8 m below the baseline (the green measurement line). The roof is still visible until couple of frames later but the contour is getting weaker as it gradually disappears behind the dust cloud.

At frame 1050 the light part of the antenna is completely visible and measured. This is the short light blue line showing a measured length of 19.4 m in Figure 23. Together with the lower part we find the measured length of these two antenna sections to be 79.8 m. The eastward angle of the antenna is 5° .

Figure 24 showes frame 1071, t = 4.57 sec. This is the last frame where the top part of the white antenna section is still visible. The light blue line of 79.8 m length indicates the position of the antenna with an assumed angle of 9° to the east. The distance from the baseline to its lowest point is therefore $1.03 \cdot 70.0 \text{ m} = 72.1 \text{ m}$. This is the point where the bottom of the roof is at this time. Of course, we assume here that the roof still exists. The antenna not only tilted eastwards but also southwards. In [NIST, 2005d, p. 166] an angle of 8° is mentioned. So there is a small additional correction factor of $\cos(8^\circ) = 0.99$, which gives a decent of $0.99 \cdot 1.03 \cdot (70.0 - 3) \text{ m} = 68.3 \text{ m}$ for the top of the roof.

If we assume the total elevation of the middle of the roof to be 417 m at collapse initiation, we can summarise the measurements in the following table, including appropriate error estimates. The error estimate for the last value (antenna based) is bigger as the antenna might not be fixed on the roof, as we have mentioned the movement of the antenna started little before t = 0.

| Time/sec | Part of roof | Distance to baseline/m | Elevation over concourse level/m |
|----------|--------------|------------------------|----------------------------------|
| 0 | top, middle | 0.7 | 417 |
| 1.64 | top, middle | 10.0 | 408 ± 2 |
| 3.20 | top, middle | 34.8 | 383 ± 2 |
| 4.57 | top, middle | 68.3 | 349 ± 4 |

TABLE 1. Results of height measurements.

2.4. The Downward Movement (Part 2). The video material of the Sauret video has already been used in [MacQueen and Szamboti 09] and [Chandler, 2010] to determine the downward acceleration of the roofline of the North Tower with shorter time intervals during the first three seconds. Their basic findings were a movement of the roof with a constant acceleration of $22.8 \text{ ft/sec}^2 = 6.95 \text{ m/sec}^2$, and 6.31 m/sec^2 , respectively. To quickly compare our results with these two we do a linear regression for a parabola $t \mapsto \frac{1}{2} a t^2 + b$. The four data points from the first

and third column of Table 1 give

(12)
$$a = 6.46 \,\mathrm{m/sec^2}, \quad b = 1.14 \,\mathrm{m}, \quad r = 0.99988,$$

where r is the regression coefficient.

The empirical data from Table 1 are illustrated in the six diagrams of Figure 4 by the horizontal black lines which indicate the error bars. The actual values in the middle are not displayed. The coloured curves are the predicted model curves for the indicated values and for the other parameters as given in (10). The parabola in the upper right corner is the one derived from (12) and displayed for reasons of comparison.



FIGURE 4. The movement of the roof (top) during the first 5 seconds.

For simplicity we do not give a sophisticated optimisation analysis here, but based on the printed model curves we take W = 250 MJ, $\lambda = 0.15$, $\kappa_{out} = 0.25$, $\kappa_e = 0.2$, $\beta = 0.05 \cdot 10^{6}$ kg/m as the result with which we continue to work. The precise values for the best fit will not be important for our main result (cp. Section 2.7). The red graph in all six diagrams shows this solution.

Note, firstly, that higher values of κ_{out} become more and more unrealistic in a gravity-driven collapse. Secondly, a higher value of β would require a lower value of W.³ Thirdly, a lower value of W would also match better if the starting time of the model curve is put slightly later at $t_{late} = \tau \sec, \tau \in [0, 0.2]$. This would be a legitimate adjustment, as the model only describes the dynamical aspect of the collapse itself. It does not model the transition from the stable to the unstable state which takes a finite time interval. In this respect the value of W = 250 MJ is only an upper bound of energy dissipation for the first $(4.6 - \tau)$ sec. This is important later.

Unfortunately, there is no reference in [Bažant and Verdure, 2007, Bažant et al. 2008] about the video footage that has been used and no indication about the starting time for their measurements. So neither we can comment on their starting time nor on the accuracy of their measurements. The discrepancy of 250 MJ and the afore mentioned value of 380 MJ in [Bažant et al. 2008] (cp. 1.3) might be mainly due to the numerical error of the too big mass of the top section.

2.5. **History Channel.** We only want to evaluate one still from a documentary aired on History Channel [History Channel]. The frame rate of this video is 59.97 frames per second, so 6 frames correspond to 0.1 seconds. This footage shows the destruction of the North Tower recorded from West Street from a north-west ground perspective. It does not show all of the collapse, as the first few seconds are missing.

Figure 5 shows the collapsing tower at frame 262. It is possible to identify the time of frame 262 with a precision of one frame in Sauret's video. This is done in Appendix B and the result is $t = (7.71 \pm 0.033)$ sec.

The building to the left is WTC 7. It was one of the three high-rise buildings that collapsed on the 11th of September 2001. It had 47 storeys and its roofline had a height of $h_1 = 610$ ft = 185.9 m [NIST, 2008, p. 5]. The green line follows the roofline. Once the camera position is known we can determine the height h_X of the point *X* that is behind the green line right on the corner of the tower. The camera position is determined in Appendix A.1. If we assume the height of the camera to be $h_0 = 1.7$ m,⁴ then the camera was located on West Street, in a distance of $d_0 = 694$ m \pm 9 m away from the north-west corner of the tower (Figure 30). The distance from the north-west corner of the to the ground (that's the bold green line in Figures 30, 29) is determined to be $d_{int} = 175$ m \pm 2 m. (That's the red line in Figures 30, 29.) Therefore the point *X* on the corner of the tower has an elevation of

(13)
$$h_X = (h_1 - h_0) \frac{d_0}{d_0 - d_{\text{int}}} + h_0$$
$$= 248 \,\text{m} \pm 2 \,\text{m}.$$

 $^{^{3}}$ The same is true for the erroneous assumption on the velocity profile of the middle section which would require a lower value of *W* as explained in [Schneider, 2017b].

⁴ Because the distance d_0 is much bigger than d_{int} , the camera height has no practical influence on the result of (13). E.g. an additional height of 2 m would reduce the height of the crushing front by $2 \text{ m} \cdot \frac{d_{int}}{d_0 - d_{int}} = 0.7 \text{ m}$ only.



FIGURE 5. The collapsing tower from West Street, frame 262 of the History Channel clip.

Apparently, Figure 5 shows that the crushing front is about to reach the point *X*. Some amount of dust is already blown outwards below *X*, but the perimeter columns are still standing without being affected. This is agreed in [Bažant et al. 2008, p. 901], where it is stated:

"Some critics believe that the bottom of the advancing dust cloud seen in the video represented the crushing front. However, this belief cannot be correct because the compressed air exiting the tower is free to expand in all directions, including the downward direction. This must have caused the dust front to move ahead of the crushing front[...]"

In other words the point *X* is only a *lower bound* for the approaching crushing front. However, we shall give an argument in Section 2.7 that the distance between *X* and the crushing front is probably small.

Now recall that the original height of the tower was 417 m and that the falling upper block had an initial height of at least 46 m. This means that at the time of frame 262 a distance of at most 417 m - 46 m - 248 m = 123 m has been crushed.

If we assume a gravity-driven collapse, then the top 46 m are still undestroyed, and a falling section of height $\lambda \cdot 123 \text{ m} + 46 \text{ m}$ sits somewhere above point X in Figure 5. We conclude that the roof had a total elevation of at least $248 \text{ m} + 46 \text{ m} + \lambda \cdot 123 \text{ m}$ above concourse level at the time of frame 262. For a compaction parameter of $\lambda = 0.15$ this gives an elevation of 312 m.

Note that if one assumes a bigger height for the initial falling section (as in [Bažant and Verdure, 2007, Bažant et al. 2008]), a bigger value of λ (as in [Bažant and Verdure, 2007, Bažant et al. 2008]) or a bigger height for the crushing front one obtains an even higher elevation of the roof.

2.6. **CBS.** We do the same routine as for the History Channel clip for a clip from CBS. The copy of the film that we use has a frame rate of 25 frames per second [CBS]. Figure 6 shows frame 739 of this clip at a time of t = 9.25 sec after collapses initiation (s. Appendix B.2 for synchronising the CBS clip). It shows that the crushing front is about to pass the point *Y* behind the green line (the roofline of WTC 7). The falling debris obstructs the view to most of the ejected dust from the tower,



FIGURE 6. The collapsing tower from West Street, frame 739 of the CBS clip.

but going through the actual video clip shows that the moment is captured correctly in the sense that this is the last moment for which we can conclude that the crushing front is above the point Y. For comparison Figure 7 shows the same clip 10 and 20 frames (0.40 sec and 0.80 sec) earlier.

In Figure 47 the distance from the north west corner to the green line in the direction of the CBS camera is measured by the dark blue line. This distance is $d'_{\text{int}} = (170 \pm 2) \text{ m}$. In Appendix A.2 we find that the distance between the CBS camera and the north-west corner is $d'_0 = (1202 \pm 20) \text{ m}$. So if we again assume a camera height of $h_0 = 1.7 \text{ m}$ we find another lower bound for the crushing front by the height h_{Υ} of the point Υ on the tower at time t = 9.25 sec:

(14)
$$h_Y = (h_1 - h_0) \frac{d'_0}{d'_0 - d'_{int}} + h_0$$
$$= 216 \,\mathrm{m} \pm 1.5 \,\mathrm{m}.$$

This estimate implies that a height of not more than 417 m - 46 m - 216 m = 155 m has been compacted. For $\lambda = 0.15$ this gives an elevation of $216 \text{ m} + 46 \text{ m} + \lambda \cdot 155 \text{ m} = 285 \text{ m}$.

2.7. **The Downward Movement (Part 3).** The error bars for the measured data are shown in Figure 8 as before by horizontal black lines. The dotted lines for the two lower measurements indicate that this is only a lower bound.

The red graph is identical in all four diagrams and shows the solution of the Crush-Down Equation with the maximal energy dissipation during the first seconds as explained in 2.4, i.e. W = 250 MJ, $\lambda = 0.15$, $\kappa_{\text{out}} = 0.25$ and the other



FIGURE 7. The collapsing tower from West Street, frames 719 (top) and 729 (bottom) of the CBS clip.

parameters as in (10). Note that the red graph misses the empirical data point at 7.71 sec by 40 m, so we detect a major discrepancy here. This discrepancy would be significantly bigger for the above discussed value of an energy dissipation of only W = 100 MJ per storey.

The graphs in other colours are also solutions of the Crush-Down Equation for the same choice of parameters except that to the upward force F_0 an extra upward

force is added over a certain interval. The interval is indicated above each diagram. It specifes the position of the roof where the force is turned on, and the position of the roof where it is turned off again.

Two types of extra forces we have used for the computations: (*a*) A constant force $F_{\text{const}}(z) = W_{\text{const}}/h$ and (*b*) an extra force F_+ that is directly proportional to F_0 by the factor W_+/W , i. e. in this case the total upward force is again proportional to F_0 , namely $F_0 + F_+ = (W+W_+)/W \cdot F_0$. Therefore the sum $W + W_+$ is the quantity which can be directly compared to the values discussed in Section 1.3. The force F_+ is the relevant quantity, as it reflects the column strength of the actual building. The discussion of the force F_{const} is given for reasons of comparison.

All extra forces are turned on 10 m above the upper error bar of the third measurement, i. e. at 363 m in all diagrams. This takes the solutions out of the measured position at time t = 4.57 sec, but we are interested in the minimal extra force that must be applied to match the two lower data points. By increasing the height where the extra force is turned on, we decrease the value of the necessary extra force to reach the lower data points at t = 7.71 sec and t = 9.25 sec, so this gives an error that decreases our result.

Three intervals are considered: Turning off at 318 m, turning off at 311 m and not turing of at all. The height 311 m is the height of the lower error bar of the measurement at 7.71 sec. The extra force is minimised if it is applied until 311 m.

The magenta graph in the upper right diagram shows that during the time interval from $t = 4.57 \sec to 7.71 \sec an additional energy of at least <math>W_{\text{const}} = 2500 \text{ MJ}$ per storey was dissipated. The minimal value for W_+ to reach the data point at $t = 7.71 \sec is W_+ = 1700 \text{ MJ}$ (the blue graph). This corresponds to an energy dissipation of $W + W_+ = 1950 \text{ MJ}$ per storey at impact height. (The blue graphs in all three diagrams have the same value of W_+ .)

The two diagrams at the bottom indicate that this value is extremely close to arresting the collapse. Indeed the collapse would arrest if this extra force would continue 10 more meters (or a little more than another second) as one can see from the blue graph in the lower right. Note that the solution with the constant extra force does not arrest if the extra force stays turned on (the magenta graph in the lower right). The increasing strength of the actual columns is responsible for this effect.

The diagram to the lower left shows that the collapse would also arrest if only a slightly bigger value of 1850 MJ would apply (the yellow graph). The yellow graph terminates within the errorbars. Therefore the distance of the crushing front to the dust front cannot be bigger than the distance from the yellow graph to the lower one of the two error bars at time t = 7.71 sec (which is less than 5 meters, i. e. less than two storeys), for otherwise the collapse would have terminated.⁵ A reasonable assumption is that the distance from the crushing front to the dust front is constant. This implies that the solution of the blue graph, which does not match the data point at t = 9.25 sec (in the upper right diagram) is not the solution that we are looking for. But if one increases the extra force a little ($W_+ = 1800$ MJ), and tuns it off earlier at 318 m, one obtains the solution given in the upper left diagram This solution satisfies the empirical requirements. This solution also seems to be a better fit because from just watching the History Channel clip one might guess that the velocity of the dust front is not decelerating when it approaches the point *X*. A more refined measurement could clarify this impression.

⁵ To be precise at this point: The distance from the dust front to the crushing front could be bigger than the 5 m-distance of the yellow solution to the lower error bar, but that would mean that an even higher extra force did occur (over a shorter interval).



FIGURE 8. The measured data and the model curves.

Note that the black graph has an energy dissipation that corresponds to an energy dissipation of $W + W_+ = 2050 \text{ MJ}$ per storey at impact level.

3. DISCUSSION OF OBSERVATIONS

3.1. The Magnitude of Energy Dissipation — Revisited. Under the principal assumption that the gravity-driven collapse model of [Bažant and Verdure, 2007, Bažant et al. 2008] describes the collapse we found that the dissipated energy due to column buckling through the first 4.6 seconds was on a scale of at most 250 MJ per storey. In the subsequent three seconds this value increased by almost an order of magnitude to over 2000 MJ. After that time period it fell back to the initial low value. (Here we refer to the values W_+ relative to the columns at impact height.)

If the maximal possible dissipation of energy per storey is on a scale below 2000 MJ (as it is demanded in [Bažant and Le (2016)]), then this implies that the principal assumption is wrong.

If the maximal possible dissipation of energy can reach the high values which we determined (as the empirical studies of Korol and Sivakumaran indicate [Korol and Sivakumaran, 2014]), then we must urgently face the question why this value was not reached during the whole of the collapse, i. e. before 4.6 seconds and after 7.7 seconds—either of which would have terminated the propagation of the collapse. Understanding the mechanism that enabled this fluctuation of energy dissipation must have priority in a thorough investigation of the collapse. In particular, there is no reason whatsoever that one should expect that the collapse was



FIGURE 9. A blink that seems to appear from the shade. Visible on frame 287 and 288 of the History Channel clip. The image shows frame 288 (cropped).

inevitable and could not have been arrested by the energy dissipation of the buckling columns at any stage during the first 8 sec of the collapse (and even later).

The numerical values for λ , z_0 and μ_0 that we used are all three smaller than the values used in [Bažant and Verdure, 2007, Bažant et al. 2008]. If we did the same analysis for the higher values therein, our result would be even more dramatic in the sense that the additional amount of energy dissipation W_+ would be bigger. (This statement is obvious for μ_0 and also for z_0 , because less height is compacted. To discuss the parameter λ note that the height of the roof as computed from the measured position of the dust front decreases with a smaller λ .)

It would be desirable to have a more refined measurement of the downward movement of the crushing front/dust front. (We only used two data points.) As we have determined the camera position for the two clips from History Channel and CBS, a detailed analysis is possible, but requires much more effort. (The camera angle is changing, and the camera is zooming simultaneously.)

3.2. Light Emissions from the Dust Cloud. To synchronise the different videos we have used in Appendix B some sort of flash-like events (blinks) in the dust cloud. There is a large number of these events (we estimate easily more than one

hundred). Many appear only for one or two frames in the video records. Because they are visible from different camera perspectives and some also seem to appear out of the shade, it is very unlikely that these are sunlight reflections. Figure 9 shows such a blink. (The two blinks shown in Figures 36 to 41 also seem to appear from the shade.) Moreover, the intensity and the rapidness of their appearance and disappearance make it unlikely that this is just light coloured building material that moves out of the dust cloud. This strongly indicates that there was a lightemitting substance in the dust.

It has been reported that the World Trade Center dust contained some sort of energetic material [Harrit et al., 2009]. There might be a connexion between theses two observations. Further research about this topic should be committed.

3.3. **Conclusion.** This work has presented fundamental empirical data of the collapse of the North Tower of the World Trade Center. These data reveal some highly remarkable phenomena during the collapse. A thorough investigation of the collapse is needed to answer the questions that compellingly arise at this stage.

APPENDIX A. DETERMINING THE CAMERA POSITION

A.1. **History Channel.** To determine the camera position we compare its perspective with the perspective of a known camera position. Other methods are applicable to determine the position, however, we present this method, because it is the most precise one we found.

Consider Figure 25. This is a still from the NBC News coverage on the 11th of September 2001. The still is taken at 48 sec [NBC]. The camera is located on the green separation line on West Street. The building to the left is the Borough of the Manhattan Community College. The visible bridge that crosses West Street approximately 100 m southwards is the Tribeca Bridge (also known as Stuyvesant Bridge). The big white building is 101 Barclay Street and the tall building behind is WTC 7. Note that the camera position is uniquely determined by the position of the two street lamps in the picture, which coincidentally happen to be in line the north-east corner of 101 Barclay Street, and the north-east corner of the top floor of the same building, which is also in line with the north west corner of WTC 7.

Figure 26 shows a Google Street View screen shot of the same location. It is dated January 2013.

Figures 27, 28 and 29 show enlarged parts of Figure 30, which is material from an aerial photograph taken in 2006 and available on the website of the City of New York [NYC, 2006]. The intersection of the blue and the cyan line in these images is the NBC camera position. In Figure 27 the street lamps are visible on the pavement. They are used for placing the blue and the cyan line.

The length calibration for Figure 30 (the white line of 315.0 m) is set between two randomly chosen street lamps which have a distance of 315.0 m. This length itself is determined by the online tool provided by the City of New York (Figure 31).

Using figure Figure 32, which is taken from [FEMA, 2002, Chap. 1, p. 1-13], we can reconstruct the north-west corner of the North Tower and the north-west corner of WTC 7 in Figure 30. These are the green and white triangles in the lower part of the images. The thick green line indicates the line following the direction of the north facade of WTC 7.

Therefore the blue line in Figure 30 measures the distance from the NBC camera position to WTC 7. This length is $d_2 = 513.1$ m. The distance d_1 of the north-west corner of WTC 7 and the north east corner of the top floor of 101 Barclay Street (the short black line) is measured to be 135.8 m. These length measurements have a precision of ± 1 m.



FIGURE 10. Comparing the two camera positions.

Now we are ready to determine the History Channel camera position by comparison. Compare Figure 33 with Figure 34. Figure 33 shows a cropped part of the NBC camera image four seconds later (and less blurred) than Figure 25). Figure 34 is a cropped part of frame 328 of the footage used by History Channel. We see that the camera positions are similar but little different. There is a tiny displacement of the History Channel record to the west and a clear displacement to the north, which is recognisable by comparing the indicated vertical measurements. The horizontal (black/white) calibration lines are set to 100 reference units. Note that the quotient of the two measured vertical distances is independent of the length of the reference unit. The measurements of the vertical lengths have an error of less than 0.5 reference units.

Because we know the height of WTC 7 ($h_1 = 610$ ft = 185.9 m [NIST, 2008, p. 5]) and the height of 101 Barclay Street ($h_2 = 99.06$ m [Emporis, 2016]), we are in the situation illustrated in Figure 10. This enables us to determine the distance d_3 by the two geometric equations

(15)
$$\frac{d_3}{h_1 - h_0} = \frac{d_1}{h_1 - h_2 - r_2}, \quad \frac{d_2}{h_1 - h_0} = \frac{d_1}{h_1 - h_2 - r_1},$$

which gives

(16)
$$d_3 = d_1 \left(1 - \frac{h_2 - h_0}{h_1 - h_0} - \frac{r_2}{r_1} \left(1 - \frac{h_2 - h_0}{h_1 - h_0} - \frac{d_1}{d_2} \right) \right)^{-1}$$

$$563\,\mathrm{m}\pm9\,\mathrm{m}.$$

With this length the position of the History Channel camera is found at the north end of the violett line in Figure 30. The distance to the north-west corner of the North Tower (the yellow line in Figure 30) is then

(17)
$$d_0 = 694 \,\mathrm{m} \pm 9 \,\mathrm{m}.$$

A.2. **CBS.** The position of the CBS camera can be determined by frame 400 as shown in Figure 44 up to an ambiguity of ± 20 m. It is located on West Street between the two intersections Desbrosses Street and Vestry Street. For comparison

Figure 45 shows the same position. A distance from the camera to the north-west corner of the North Tower of (1202 ± 20) m is measured in Figure 46.

APPENDIX B. SYNCHRONISING THE VIDEO CLIPS

B.1. **History Channel.** There are some rough methods to pre-adjust the Sauret clips and the History Channel clip up to a third of a second. E. g. there is a clearly recognisable black part of debris falling left (east) to the tower, which is right on the first frame of the History Channel video. In the Sauret Video this very same piece of debris is visible from approximately $t = 3.2 \sec$ to $t = 3.7 \sec$ in the lower left.

Once a rough calibration is done we are looking for an event that can be used to a synchronisation up to one frame. There are plenty of those: In the dust there are numerous and well localisable 'blinks' appearing. Some of them are only visible for one frame.

Not all of the blinks are visible in all camera perspectives, but we use such one for synchronisation that is visible in at least three records. The third record we use is taken from a CNN documentary [CNN] and gives an intermediate perspective between Sauret's camera and the camera on West Street. Figureres 35 to 41 show the disappearance of the same blink in the three perspectives from one frame to the next. This is 1089 to 1090 in Sauret's video, 110 to 111 in the History Channel video and 127 to 128 in the CNN video. We therefore identify the timeline of these videos at this step. For controlling reasons we have verified this synchronisation with other blinks and found confirmation up to one frame. This is as good as it can possibly be. Note the History Channel clip has the double frame rate of Sauret's video.

Consequently, the time of frame 262 of the History Channel clip is given by the time of frame 1165 ± 1 in Sauret's video. This time is $t = 7.71 \sec \pm 0.03 \sec$ after collapse initiation (at frame 934).

B.2. **CBS.** The blink that has been used to synchronise the Sauret video and the History Channel clip is not clearly visible in the CBS clip. But the appearance of another blink (Blink 2) in Figures 39 to 42 can be used to synchronise the History Channel clip and the CBS clip: The step from frame 636 to 637 (CBS) corresponds to 109 to 110 (History Channel), which happens at time t = 5.17.

The frame rate of the CBS clip is 25 frames per second. Therefore fame 739 of the CBS clip is 4.08 sec after frame 637. This is t = 9.25 sec after collapse initiation.

APPENDIX C. COMPUTING NUMERICAL SOLUTIONS WITH MAXIMA

The following is the source code which we have used to compute the solutions of the Crush-Down Equation with Maxima [Maxi]. The variable z_1 corresponds to 29 *h*. v_0:0 sets the initial velocity to zero. Note for the computation that the mass density μ_0 , the parameter β and the energy absorption capacity of the columns *W* miss a factor 10⁶ in the source code. However, this factor cancels out in the coefficients ϕ and ψ , so the solution is not effected by this simplification.

```
/* [wxMaxima: input start] */
/* [Define the constants] */
mu_0:0.6; g:9.8; H:417; h:3.8; z_0:46; z_1:110;v_0:0;
```

/* [We compute 4 soltuions so we give the following parameters fourfold] */

lambda_1:0.15;kappa_1:0.25;

lambda_2:0.15;kappa_2:0.25; lambda_3:0.15;kappa_3:0.25; lambda_4:0.15;kappa_4:0.25;

alpha_1:0.1*kappa_1*(1-lambda_1)^2; beta_1:0.05; alpha_2:0.1*kappa_2*(1-lambda_2)^2; beta_2:0.05; alpha_3:0.1*kappa_3*(1-lambda_3)^2; beta_3:0.05; alpha_4:0.1*kappa_4*(1-lambda_4)^2; beta_4:0.05;

W_1:250; W_2:250; W_3:250; W_4:250;

/* [The measured data] */

t.1:1.64; a.1:408; error_1:2; t_2:3.20; a.2:383; error_2:2; t_3:4.57; a.3:349; error_3:4; t_4:7.71; a.4:248; error_4:2; t_5:9.25; a.5:216; error_5:1.5;

/* [The Heaviside step function] */

theta(z):=if z<0 then 0 else 1;</pre>

/* [The damage function] */

 $chi(z) := (0.5+0.4*theta(z-z_0-h)+0.1*theta(z-z_0-4*h));$

/* [The mass density and the mass function] */

mu(z):= mu_0*(1+theta(z-z_1)*(0.43*(z-z_1)/(H-z_1)));

m.1(z):= mu_0*z_0+ (1-kappa_1)*mu_0*(z-z_0+ theta(z-z_1)*0.215*(z-z_1)^2/(H-z_1)); m.2(z):= mu_0*z_0+ (1-kappa_2)*mu_0*(z-z_0+ theta(z-z_1)*0.215*(z-z_1)^2/(H-z_1)); m_3(z):= mu_0*z_0+ (1-kappa_3)*mu_0*(z-z_0+ theta(z-z_1)*0.215*(z-z_1)^2/(H-z_1)); m_4(z):= mu_0*z_0+ (1-kappa_4)*mu_0*(z-z_0+ theta(z-z_1)*0.215*(z-z_1)^2/(H-z_1));

/* [The amount of extra energy dissipation] */

W_extra_1:1000; W_extra_2:1500; W_extra_3:2000;

/* [Turning the forces On and Off] */

on:a_3+error_3+10; off:a_4-error_4+z_0+lambda_1*(H-z_0-(a_4-error_4));

On_1(z):=theta(z-(z_0+(H-on)/(1-lambda_1))); Off_1(z):=-theta(z-(z_0+(H-off)/(1-lambda_1))); On_2(z):=theta(z-(z_0+(H-on)/(1-lambda_2))); Off_2(z):=-theta(z-(z_0+(H-off)/(1-lambda_2))); On_3(z):=theta(z-(z_0+(H-on)/(1-lambda_3))); Off_3(z):=-theta(z-(z_0+(H-off)/(1-lambda_3)));

/* [The extra forces] */

Extra_1(z):= W_extra_1/h * (On_1(z)+Off_1(z)); Extra_2(z):= W_extra_2/h * (On_2(z)+Off_2(z));

```
Extra_3(z):= W_extra_3/h * (On_3(z)+Off_3(z));
```

/* [The total upward column force] */

```
F_1(z):= (Extra_1(z)+W_1/h)*(1+theta(z-z_1)*(6*(z-z_1)/(H-z_1)));
F_2(z):= (Extra_2(z)+W_2/h)*(1+theta(z-z_1)*(6*(z-z_1)/(H-z_1)));
F_3(z):= (Extra_3(z)+W_3/h)*(1+theta(z-z_1)*(6*(z-z_1)/(H-z_1)));
F_4(z):= (W_4/h)*(1+theta(z-z_1)*(6*(z-z_1)/(H-z_1)));
```

/* [The coefficients of the Crush-Down Equation] */

```
phi.1(z):=g/(1-lambda.1)-chi(z)*F.1(z)/((1-lambda.1)*m.1(z));
phi.2(z):=g/(1-lambda.2)-chi(z)*F.2(z)/((1-lambda.2)*m.2(z));
phi.3(z):=g/(1-lambda.3)-chi(z)*F.3(z)/((1-lambda.3)*m.3(z));
phi.4(z):=g/(1-lambda.4)-chi(z)*F.4(z)/((1-lambda.3)*m.4(z));
```

```
psi_1(z):=(1-kappa_1)*mu(z)/m_1(z) + (alpha_1*mu(z)+beta_1)/((1-lambda_1)*m_1(z));
psi_2(z):=(1-kappa_2)*mu(z)/m_2(z) + (alpha_2*mu(z)+beta_2)/((1-lambda_2)*m_2(z));
psi_3(z):=(1-kappa_3)*mu(z)/m_3(z) + (alpha_3*mu(z)+beta_3)/((1-lambda_3)*m_3(z));
psi_4(z):=(1-kappa_4)*mu(z)/m_4(z) + (alpha_4*mu(z)+beta_4)/((1-lambda_4)*m_4(z));
```

/* [Compute the solutions with Runge-Kutta] */
/* [Using the Heaviside function the solutions arrest at negative velocities] */

time:12;stepwidth:0.01;

```
solution_1:rk ( [u* theta(u), phi_1(z)-u^2*psi_1(z)], [z, u], [z_0,v_0], [t,0,time,stepwidth])$
solution_2:rk ( [u* theta(u), phi_2(z)-u^2*psi_2(z)], [z, u], [z_0,v_0], [t,0,time,stepwidth])$
solution_3:rk ( [u* theta(u), phi_3(z)-u^2*psi_3(z)], [z, u], [z_0,v_0], [t,0,time,stepwidth])$
solution_4:rk ( [u* theta(u), phi_4(z)-u^2*psi_4(z)], [z, u], [z_0,v_0], [t,0,time,stepwidth])$
```

/* [Turn the solutions into the height of the roof] */

height_1:makelist([solution_1[i][1],H-(1-lambda_1)*(solution_1[i][2]-z_0)],i,1,length(solution_1))\$
height_2:makelist([solution_2[i][1],H-(1-lambda_2)*(solution_2[i][2]-z_0)],i,1,length(solution_2))\$
height_3:makelist([solution_3[i][1],H-(1-lambda_3)*(solution_3[i][2]-z_0)],i,1,length(solution_3))\$
height_4:makelist([solution_4[i][1],H-(1-lambda_4)*(solution_4[i][2]-z_0)],i,1,length(solution_4))\$

/* [Plot the solutions and the empirical data] $\ast/$

```
wxplot2d( [
[discrete,height_1], [discrete,height_2], [discrete,height_3], [discrete,height_4],
```

[parametric, t_1, t, [t, 0, a_1]], [parametric,t, a_1+error_1, [t, 0,t_1]], [parametric,t, a_1-error_2, [t, 0,t_1]],

[parametric, t.2, t, [t, 0, a.2]], [parametric,t, a.2+error.2, [t, 0,t.2]], [parametric,t, a.2-error.2, [t, 0,t.2]],

[parametric, t_3, t, [t, 0, a_3]],
[parametric,t, a_3+error_3, [t, 0,t_3]],

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[parametric,t, a_3-error_3, [t, 0,t_3]],

```
[parametric, t.4, t, [t, 0, a.4+z.0+lambda.1*(H-z.0-a.4)]],
[parametric,t, a.4-error.4+z.0+lambda.1*(H-z.0-(a.4-error.4)),[t, 0,t.4]],
[parametric,t, a.4+error.4+z_0+lambda.1*(H-z.0-(a.4+error.4)),[t, 0,t.4]],
```

[parametric, t.5, t, [t, 0, a.5+z.0+lambda_1*(H-z_0-a.5)]], [parametric,t, a.5-error_5+z_0+lambda_1*(H-z_0-(a.5-error_5)),[t, 0,t.5]], [parametric,t, a_5+error_5+z_0+lambda_1*(H-z_0-(a_5+error_5)),[t, 0,t.5]]]],

[x,0,time], [y,260,420],

/* [wxMaxima: input end] */

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FIGURE 11. Horizontal calibration measurement at frame 800, t = -4.47 sec.



FIGURE 12. Distance from 145 Avenue of the Americas to the WTC complex based on an aerial photograph from 1996. The distance measurement is done with the provided online tool of the City of New York [NYC, 1996]. Note for the angle measurement that the optical center of the camera points little eastwards to the building.



FIGURE 13. Screenshot from Google Street View showing 101 Avenue of the Americas and 145 Avenue of the Americas. [Google, 2017].

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FIGURE 14. Measuring a known vertical distance, t = -4.47 sec.



FIGURE 15. North side of the North Tower, [NIST, 2005c, p.35]. The white lines and the floor number are added on the basis of Figure 16.



FIGURE 16. Schematic illustration of the aircraft impact zone [NIST, 2005a, p. 22].



FIGURE 17. Structural drawing of the North Tower [NIST, 2005b, p. 18].



FIGURE 18. Measuring the position of the bottom of the roof at frame 934, t = 0 sec.



FIGURE 19. Measuring the height of the initially falling block at frame 957, t = 0.77 sec.



FIGURE 20. Measuring the position of the bottom of the roof at frame 983, t = 1.64 sec.



FIGURE 21. Measuring the lower part of the antenna at frame 1024, t = 3.00 sec.



FIGURE 22. Measuring the position of the bottom of the roof at frame 1030, t = 3.20 sec.



FIGURE 23. Measuring the white part of the antenna at frame 1050, t = 3.87 sec.

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FIGURE 24. Reconstructing the position of the bottom of the roof at frame 1071, t = 4.57 sec.



FIGURE 25. The NBC News camera perspective at 48 sec [NBC].



FIGURE 26. Screen shot from Google Street View, showing West Street in January 2013, [Google, 2013].



FIGURE 27. Zoom into the top part of Figure 30.



FIGURE 28. Zoom into the upper part of Figure 30.



FIGURE 29. Zoom into the lower part of Figure 30.



FIGURE 30. Aerial photograph of NYC, dated 2006, [NYC, 2006]. The green and white measurements at the bottom indicate the reconstruction of the position of the NW corner of the North Tower and of the north face of WTC 7 (cp. Figure 32). After having reconstructed the the position of the NW corner we noticed the clearly recognisable ground formation that forms a right angle where we determined the corner (cp. Figure 29). This might be remains of the actual foot print of the tower, which indicates that the reconstruction is done properly.



FIGURE 31. Calibration measurements for Figure 30 using the online measurement tool of [NYC, 2006].

FIGURE 32. Aerial photograph, taken from [FEMA, 2002, Ch. 1, p. 1-13]. Using the displayed measurements one can reconstruct the position of the north west corner of the North Tower and the north side of WTC 7 in Figure 30 by just transporting all triangles.

FIGURE 33. NBC News camera perspective (cropped) at 52 sec.

FIGURE 34. History Channel camera perspective (cropped), frame 328.

FIGURE 35. Zoom into frame 1089 of Sauret's video.

FIGURE 36. Zoom into frame 1090 of Sauret's video.

FIGURE 37. For comparison: Zoom into frame 127 of the CNN video.

FIGURE 38. For comparison: Zoom into frame 128 of the CNN video.

FIGURE 39. Zoom into frame 109 of the History Channel video.

FIGURE 40. Zoom into frame 110 of the History Channel video.

FIGURE 41. Zoom into frame 111 of the History Channel video.

FIGURE 42. Frame 636 of the CBS clip.

FIGURE 43. Frame 637 of the CBS clip.

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FIGURE 44. Frame 400 of the CBS clip [CBS].

FIGURE 45. Screen shot from Google Street View, showing West Street in October 2016, [Google, 2016].

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FIGURE 46. Distance from the CBS camera position to the WTC complex based on an aerial photograph from 1996. The measurement is done with the provided online tool of the City of New York [NYC, 1996].

FIGURE 47. The dark blue line indicates the distance from the north-west corner of the tower to the intersection of the green line in the direction of the CBS camera.