

# ENERGY ESTIMATES OF PROGRESSIVE FLOOR COLLAPSES AND THE WORLD TRADE CENTER CATASTROPHE

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**ABSTRACT.** The Simple Collapse Model of Bažant and Zhou is evaluated for a progressive floor collapse of a tall building. A sequence of energy estimates indexed by the collapsing floors is derived. Each of the estimates gives a sufficient condition to arrest the collapse at a given floor.

The first estimate of this sequence has been stated by Bažant and Zhou and has been repeatedly cited later on. However, this estimate is not optimal in the sense that the following estimates give a weaker condition to arrest the collapse.

**Keywords:** Progressive Floor Collapse, Structural Dynamics, High-Rise Buildings, World Trade Center, North Tower, New York City, Terrorism.

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## 1. INTRODUCTION

On the 11th of September 2001 both the North and the South Tower of the World Trade Center in New York City, USA, collapsed after both of them were struck by an aircraft. In this short note we shall solely focus on the collapse of the North Tower. The building had 110 storeys and a roof height of 417 m. The fuselage of one of the two aircrafts impacted the North Tower at the height of the 96th floor. The Tower collapsed 102 minutes later [NIST1].

Two days after the attacks Bažant and Zhou submitted a paper to explain why the collapse might have occurred after the (total) failure of just one storey [BaZh02]. A gravity-driven progressive floor collapse is proposed as an explanation. This analysis is based on their Simple Collapse Model where the falling top section of the building descends through one storey in free fall and

impacts the floor below inelastically. Their main result was a sufficient condition for the energy absorption capacity of the buckling columns in the floor below to arrest the fall. Based on the numerical values they had at the time they found this condition was violated by almost one order of magnitude (a factor 8.4) for the North Tower and concluded that the collapse became inevitable once the failure of only one storey appeared.

This result has been repeatedly stated in the following works of Bažant and Verdure [BaVe07] and of Bažant, Le, Greening and Benson [BLGB08] and also in [NIST1-6, p. 323] without being corrected. Indeed, two things need an explicit clarification:

- (1) The numerical values of the quantities that are used for evaluating the estimate numerically. In particular these are the mass of the falling top section, the maximal possible energy dissipation of the buckling columns, and the resistance through the first failing storey.
- (2) The estimate given in [BaZh02] does not give the minimal value of energy dissipation that would be sufficient to arrest the collapse. The minimal value is lower.

The numerical values have been corrected by others, and we shall recapitulate some of the discussion below. The main objective of this short note is to explain the second one of the two mentioned points.

## 2. SIMPLE COLLAPSE MODEL

Let us carefully review the Simple Collapse Model proposed in [BaZh02] and clarify the critical condition for a global collapse.

**2.1. Model Assumptions.** We consider a collapse sequence whose beginning is illustrated in Figure 1. The top section (of mass  $M$ ) of a tall building crushes on the lower part due to column failure in a certain storey. Let us first assume a worst case scenario in which the top section of the building encounters no resistance during the fall through the storey that initially failed. We assume the following sequence of events and properties:

- All storeys have the same height  $h$ . All floors have the same mass  $m$ , but no vertical extension. (Throughout this short note we use the word ‘floor’ for the massive bottom part of each storey.)
- All columns of the building are massless and once they are broken they don’t contribute to the collapse any more.
- The columns in each storey have the same strength. So the amount of energy  $E_{\text{abs}}$  that is needed for bending/crushing the columns in one storey is the same for all storeys.

- The crushing front propagates from top to bottom in the following sense. The impact on the floor at position  $x = 0$  in Figure 1 is assumed to be inelastic. After impact the top section and that floor move downwards as a single block of aggregated mass  $M + m$ . If the energy dissipation of the buckling columns is too low to arrest the fall, the falling section impacts the floor at position  $x = h$ . This impact is again inelastic and results in an aggregated falling mass of  $M + 2m$ . Then the propagation of the collapse might continue as described.

**2.2. Remark on the Model Assumptions.** The assumption that the top section is not destroyed during the collapse is based on the argument given in [BLGB08, Appendix] where a two-sided front propagation is computed in a similar set-up. Therein the upward-directed front terminates right after collapse initiation.

**2.3. Critical Energy Absorption Capacity.** Let us denote by  $v_0$  the velocity of the top block of mass  $M$  right before impact on the first floor at position  $x = 0$  in Figure 1. By conservation of energy it satisfies

$$(1) \quad \frac{1}{2} M v_0^2 = Mgh,$$

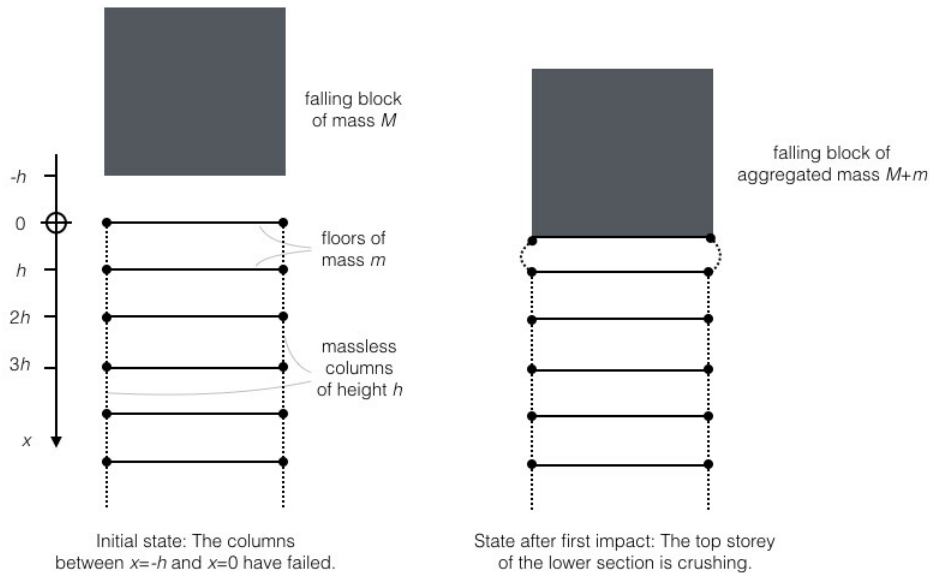


FIGURE 1. Simple Collapse Model

where  $g$  is the acceleration of gravity and  $h$  is the height of one storey. After impact on the first floor of mass  $m$  the top section and the first floor move downwards as a single block of aggregated mass  $M + m$ . The velocity right after impact can be computed by conservation of momentum and is given by

$$(2) \quad u_0 = \frac{M}{M+m}v_0 = \frac{a_0}{a_1}v_0,$$

where here and in the following we use the convention  $a_k := 1 + k\frac{m}{M}$ , for any number  $k = 0, 1, 2, 3, \dots$ . The motion of the block with mass  $M + m = Ma_1$  will stop before reaching position  $x = h$  if bending and crushing of the columns between  $x = 0$  and  $x = h$  absorbs more energy as given by the kinetic energy of the mass  $Ma_1$  and its loss of potential energy. In other words, if the energy absorption capacity  $E_{\text{abs}}$  of the columns of one storey satisfies the inequality

$$(3) \quad E_{\text{abs}} \geq \frac{1}{2}(Ma_1)u_0^2 + (Ma_1)gh = Mgh\frac{1+a_1^2}{a_1},$$

then the fall will arrest before the position  $x = h$  is reached. However, if inequality (3) is violated, then the collapse might progress. (This is the case if the maximal resistance force of the columns between  $x = h$  and  $x = 2h$  is reached during the impact. The columns will then deform plastically and will eventually buckle if the load is too big.) If the collapse progresses, then the velocity  $v_1$  of the aggregated mass  $Ma_1$  right before hitting the next floor can again be computed by energy conservation which takes the form

$$(4) \quad \frac{1}{2}Ma_1v_1^2 = Mgh\frac{1+a_1^2}{a_1} - E_{\text{abs}}.$$

Then again the impact on the floor at position  $x = h$  will be assumed to be inelastic and the falling block will have mass  $M + 2m = Ma_2$ . Conservation of momentum implies that the velocity  $u_1$  right after the impact is given by

$$(5) \quad u_1 = \frac{M+m}{M+2m}v_1 = \frac{a_1}{a_2}v_1.$$

Similar as above we conclude that the motion of the newly aggregated block with mass  $Ma_2$  will arrest before position  $x = 2h$  if the inequality

$$(6) \quad \begin{aligned} E_{\text{abs}} &\geq \frac{1}{2}(Ma_2)u_1^2 + (Ma_2)gh \\ &= \frac{a_1}{a_2} \cdot \frac{1}{2}Ma_1v_1^2 + Mgha_2 \end{aligned}$$

holds. Inserting (4) and solving the resulting inequality for  $E_{\text{abs}}$  gives

$$(7) \quad E_{\text{abs}} \geq Mgh \frac{1 + a_1^2 + a_2^2}{a_1 + a_2}.$$

Note that if the fraction  $m/M$  is small then the fraction  $(1 + a_1^2)/a_2$  is approximately 2, whereas the fraction  $(1 + a_1^2 + a_2^2)/(a_1 + a_2)$  is approximately  $3/2$ , so condition (7) is indeed weaker than condition (3).

It is left to the reader to repeat the arguments from above in an easy induction by  $n$  to conclude that the inequality

$$(8) \quad E_{\text{abs}} \geq Mgh \frac{1 + \sum_{k=1}^n a_k^2}{\sum_{k=1}^n a_k}.$$

is a sufficient condition to arrest the fall before position  $x = nh$ . It is clearly not a necessary condition, because the actual column forces are not reflected in this consideration.

The first one of these estimates ( $n = 1$ ) has been mentioned in [BaZh02, BaVe07, BLGB08], where on its basis it is argued that a gravity-driven progressive floor collapse of the Twin Towers was inevitable (cp. Section 3.3).

**2.4. Evaluation of Critical Capacity.** Using the summation formulas for the numbers from 1 to  $n$  and for the square numbers from 1 to  $n^2$  the fraction of the right hand side can be made explicit:

$$(9) \quad \frac{1 + \sum_{k=1}^n a_k^2}{\sum_{k=1}^n a_k} = \underbrace{\frac{1 + n + n(n+1)\frac{m}{M} + \frac{1}{6}n(n+1)(2n+1)\left(\frac{m}{M}\right)^2}{n + \frac{1}{2}n(n+1)\frac{m}{M}}}_{=: F\left(n, \frac{m}{M}\right)}.$$

The collapse of the North Tower of the World Trade Center originated at the 98th floor [NIST1, p.151]. The fuselage of the aircraft crushed into the 96th floor. The building had 110 storeys, so there were 12 or 13 storeys (or maybe up to 15, see the discussion about Figure 19 in [Schn17a, Section 2.3]) plus the roof with its antenna above the failing floor. We therefore consider  $0.077 \simeq 1/13$  (or  $0.063 \simeq 1/16$ ) to be a reasonable value of the fraction  $m/M$ . For  $n = 1, 2, \dots, 10$  the corresponding values of  $F(n, m/M)$  are given in Table 1.

$n$	1	2	3	4	5	6	7	8	9	10
$F(n, 0.077)$	2.01	1.56	1.45	1.41	1.40	1.41	1.44	1.46	1.49	1.53
$F(n, 0.063)$	2.00	1.55	1.42	1.38	1.36	1.37	1.38	1.40	1.42	1.45

TABLE 1. Some values of  $F$ .

The minimal value of  $F(n, 0.077)$  is 1.40 for  $n = 5$ . In the Simple Collapse Model this means: If the top section has 12 to 13 storeys, then a floor-wise energy absorption capacity that is 40% higher than the initial kinetic energy gained during the fall through the initial failing storey is sufficient to arrest the fall (before position  $x = 5h$ ).

**2.5. Model Refinement.** It is reasonable to assume the initially failing storey does absorb some energy. So the initial kinetic energy might be given as

$$(10) \quad \frac{1}{2}Mv_0^2 = \alpha Mgh,$$

where  $\alpha \in [0, 1]$ . It is demanded in [BaLe11] that  $\alpha$  should be at least 0.794. However, actual measurements of the acceleration of the roofline of the North Tower show that a lower value actually appeared [MaSz09, Chan10]. During the first 1.2 seconds, i. e. during the fall through the first storey the acceleration was 0.52 g [SSJ13, Figure 1]. Anticipating this empirical datum, the computation done in Section 2.3 can be redone exactly as before but with (1) replaced by (10). The resulting energy estimate then becomes

$$(11) \quad E_{\text{abs}} \geq MghF_\alpha \left( n, \frac{m}{M} \right),$$

where

$$(12) \quad F_\alpha \left( n, \frac{m}{M} \right) := \frac{\alpha + \sum_{k=1}^n a_k^2}{\sum_{k=1}^n a_k}.$$

For  $\alpha = 0.52$  and  $\alpha = 0.79$  we find the first ten values of  $F_\alpha(n, 0.077)$  in Table 2.

$n$	1	2	3	4	5	6	7	8	9	10
$F_{0.79}(n, 0.077)$	1.81	1.47	1.39	1.36	1.37	1.39	1.41	1.44	1.48	1.51
$F_{0.52}(n, 0.077)$	1.56	1.35	1.31	1.31	1.33	1.35	1.38	1.42	1.46	1.49

TABLE 2. Some values of  $F_{0.52}$  and  $F_{0.79}$ .

In [BaVe07] the compaction parameter  $\lambda$  is introduced to describe the size of the crushed building part relative to its original size. A value of  $\lambda = 0.18$  is used in [BaVe07] a value of  $\lambda = 0.15$  is used in [Schn17a]. We can easily include this parameter in the Simple Collapse Model by giving an extension of  $\lambda h$  to each floor. In other words, if one storey is crushed the falling section of the building has descended by a height of  $(1 - \lambda)h$ . (The impact of two floors

is still assumed to be inelastic.) Therefore the energy estimates we obtain are just the same as before but rescaled by the factor  $1 - \lambda$ .

Including  $\alpha$  and  $\lambda$  a sufficient condition to arrest the fall is given by

$$(13) \quad E_{\text{abs}} \geq Mgh(1 - \lambda)F_{\alpha} \left( n, \frac{m}{M} \right).$$

### 3. DISCUSSION OF RESULTS

**3.1. Numerical Evaluation.** In [BaZh02] a mass of  $M = 58 \cdot 10^6$  kg is stated without reference. Two related quantities that are also stated without reference in [BaVe07, BLGB08] are the total mass of the North Tower, 500,000 t, and the height of the falling top section, 80 m.

It has already been pointed out in [SSJ13] that  $M = 58 \cdot 10^6$  kg is too big and a value of  $M = 33 \cdot 10^6$  kg has been used based on [NIST1-6D, Table 4-7].

According to [NIST1, p. 151] and [Schn17a, Sec. 2.3] the falling top section had a height of 46 to 55 m. In [Schn17a] a mass density of  $0.6 \cdot 10^6$  kg/m is used for the top 30 storeys. This value is based on a total mass of 288,000 t of the tower, which is the value that has been estimated in [Uric07]. A height of 50 m (approx. 13 storeys with a height of  $h = 3.8$  m) for the top section then gives  $M = 30 \cdot 10^6$  kg.

So if we evaluate (13) with the the bigger value of  $M$  we find that

$$(14) \quad \begin{aligned} E_{\text{abs}} &\geq 33 \cdot 10^6 \text{kg} \cdot 9.8 \text{ m/sec}^2 \cdot 3.8 \text{ m} \cdot (1 - 0.15) \cdot 1.31 \\ &= 1370 \text{ MJ} \end{aligned}$$

is a sufficient condition to arrest the fall. Note that for  $\alpha = 1$  the result is just 1460 MJ, which is less then 7% bigger than the value of (14).

**3.2. Empirical Values of Energy Dissipation.** In [BaZh02] a maximal energy dissipation per storey of 500 MJ is stated. This value is based on computations for a three-hinge buckling model for the crushing columns. However, Korol and Sivakumaran have conducted empirical studies of buckling columns and found that this value must be corrected by a factor 3 to 4 [KoSi14]. This means a range from 1500 MJ to 2000 MJ should be regarded as a maximal possible value of energy dissipation per storey. As this range exceeds the value of (14), the conclusion that the collapse was inevitable is wrong if it is based on the Simple Collapse Model. For the higher value of 2000 MJ this is still correct if a structural damage of the columns of over 25% is assumed. (This statement is also correct for  $\alpha = 1$ .)

In [BaLe16] it is demanded that the empirical values of [KoSi14] should be rescaled by a factor of 2/3 which would give a range of 1000 MJ to 1300 MJ. If this range is correct, no definite statement can be made within the uncertainty

of the other relevant quantities. So it is certainly false to claim that a priori the design of the columns was too weak to arrest the fall.

It should be emphasised at this point that so far we are discussing the maximal possible amount of energy dissipation per storey. This value appears if all columns buckle according to the three-hinge model. However, this value does not match the observed values during the actual collapse. During the first 4.6 seconds of the collapse the energy dissipation per storey was below 250 MJ [Schn17a, Section 2.4, Figure 4]. Moreover, between 4.6 and 7.7 seconds after collapse initiation a value that corresponds to (at least) 2000 MJ of energy dissipation per storey must have occurred if the collapse was gravity-driven [Schn17a, Section 2.7, Figure 8]. The observation of this high value of energy dissipation implies that *if* the maximal possible of energy dissipation was only in a range of 1000 MJ to 1300 MJ, then the collapse was not gravity-driven.<sup>1</sup>

**3.3. Conclusion.** In [BaZh02] a non-optimal estimate ( $n=1$ ) was evaluated with incorrect numerical values. This led the authors to the conclusion that the energy absorption capacity of the buckling columns of the North Tower was too low to arrest the initiated collapse of the building by a factor of 8.4. Based on this erroneous result it is stated in [BLGB08]:

*“Merely to be convinced of the inevitability of [a] gravity[-]driven progressive collapse, further analysis is, for a structural engineer, superfluous. Further analysis is nevertheless needed to dispel false myths, and to acquire [a] full understanding that would allow assessing the danger of [a] progressive collapse in other situations.”*

We hope that the structural engineer to whom this statement refers is not involved in any real-life construction, for he might show a lack of critical thought in other situations, too. Yet much more than we are concerned about that one structural engineer, we are concerned about the dogmatic attitude that is manifest in these formulations.

Nonetheless we very much agree with the second one of these three propositions, to which we devote this work in accordance with its principal theme.

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<sup>1</sup> Here we use the term “gravity-driven collapse” as a shorthand for a collapse that is described by the Crush-Down equation, which essentially is a continuous (non-discrete) version of the presented Simple Collapse Model. It might be regarded as the limit  $h \rightarrow 0$  with constant total mass and total height of the building. The Crush-Down equation has been proposed and modified in [BaVe07, BLGB08]. A discussion and correction of some of the terms is given in [Schn17a] and [Schn17b].



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